George Adams

UNIVERSAL FORCES IN MECHANICS

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1. Introduction. Warmth and Inner Forces.

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The following is a resumé in as brief as possible form of the approach which I presented as a report during the Math.-Astron. Arbeitswoche at the Goetheanum in January of this year. First I should like to refer, with regard to the etheric in the mineral kingdom, to that which I stated at the beginning of my contribution "Wärmeausdehnung im Aetherraum" in No. 2 of this Korrespondenz, middle of p. 3 [transl. as "Warmth Expansion in Etheric Space" in No. 1 of the English-language Correspondence, Fall, 1972, also middle of p. 3], as principal "thesis". This work is based on the same thesis. I assume, then, that everywhere in the material world "seed-" or "germinating-points" of ether-spaces may be present. This idea proves itself to be especially fruitful in a place where one at first might least expect it: in the effects of forces in mechanics. It turns out, namely, to be quite meaningful to interpret forces and general force-like vectors (e.g. impulse-) as vectors in counterspace. From this, there arise thought-pictures which work more stimulatingly on knowing-experience than the old time-honoured ones. By naturally retaining the accustomed spatial representation for geometrical - kinematical vectors (displacement, velocity, acceleration), one is then confronted with two kinds of mutually polar entities in one's thought-picture – precisely with spatial and counterspatial vectors. Now Rudolf Steiner has indicated (right at the beginning of the First Natural Science Course) that we should become aware in mechanics of the fundamental difference between quantities of the one and the other kind; he takes his departure, as you recall, from the classical example of the parallelogram of motions and of forces. Over against the former and latter we are in completely different situations of consciousness; really natural science only begins with mechanics. I say now: already in the realm of Nature, with which we are here concerned, the universal-force-like is at work as well. There is, after all, in the sense of Rudolf Steiner, nothing in Nature – not even anything in the so-called realm of mechanics – where universal forces are not also at play.

That just these mechanical forces – that is, the ones one would suppose to be the centric forces k a t' e x o c h e n [par excellence] – also have this other aspect, I think I can make precise out of known scientific facts. If one tries to give oneself an account, namely, of the balance of forces in any mechanical system, then one always finds it necessary to take into it some at first unknown, unseen factor. As a simple example: the increase in hydrostatic pressure with depth. One draws mentally a little cylinder into the material medium and says to oneself: This must be, as a whole, in balance under the influence of all the forces involved. Result: $(p_1 - p_0)a = mg$, where a denotes the area of the cross section and mg the weight of the cylindrical volume filled with matter; etc. The thought-process is logical, in its way, but while one is carrying it out the question arises: Why don't the opposing forces – p_1a from below, p_0a from above resp. the horizontal pressures cancelling one another – crush the matter contained within this portion of space? Answer in the sense of classical physics: The "intramolecular forces" are at work there against the external forces. That is: recourse is had to something unknown, unseen, and done so with inner necessity. (I believe it may be correctly claimed even with regard to present-day physics that the same necessity is transplanted, as it were, ever further inward. If one believes to have gained insight into atomic and molecular forces by means of outer electrons sustaining the spatial structure, then there arises the question: What sort of unknown forces hold the atom's nucleus together; etc.)

The "inner necessity" that arises with this, at least for the way of thinking of classical physics, has to do – it seems to me – with the "intentional relationship" of which Rudolf Steiner writes in his book Von Seelenrätseln (1917 edition, pp. 128-130) [cf. Chapt. VIII of The Case for Anthroposophy – selections from Von Seelenrätseln, chosen, translated, and arranged by Owen Barfield, Rudolf Steiner Press, London, 1970], tying in with the psychology of Franz Brentano, that it is also necessary to look at the physical from this point of view. "All properties of the physical world are through the relationships of the things to one another ... ." Something physical, says Rudolf Steiner, is "of such a nature, that it is what it is through the relationship of something external to it". This holds for all forming of concepts in physics. As long as one thinks only physically, that is to say essentially extensive-spatially, there remains always the question: How does that which suffers the external effects, in turn exerting effects on its surroundings, in itself endure? It lies in the nature of the thing that one cannot answer this question with the same kind of consideration, but can rather at most, by forming hypotheses, shift it further inward.

But Nature herself gives us a clue to what is going on here. In all mechanical systems, be they at rest, be they in motion, elastic forces are involved, of which the hydrostatic pressure mentioned above is only the simplest example. The so-called "completely rigid body" is of course a fiction. Every architect, every engineer, knows that he must take into account the inner ductility and tensile strength of his materials, the moments of resilience and elastic recoil of a beam, etc. At rest as in motion, the elastic forces must maintain dynamic balance; if they do not, there are disasters. Now with every displacement of elastic balance, however small, shades of warmth arise. These are always present and must be taken into practical consideration as soon as one follows the process not just purely mechanically but thermodynamically. There are then exact mathematical, subtle relationships between the shades of warmth and the processes' potential for performing work, as expressed for example in the Gibbs-Helmholtz equation. If we now recognise warmth, in the sense of Rudolf Steiner, as "intensive movement" – as revelation of the dynamical interplay of space and counterspace – then it becomes clear to us: The elastic resistant forces of matter have to do with something counterspatial, i.e. ethereal. Out of heavenly forces of the entire cosmos the particular piece of matter is formed; the heavenly forces are enchanted within it, – they are certainly not extinguished, or else it would fall apart, into dust. This ethereal – this "light" enchanted into darkness – is just that element of Nature which man must take with him into even the most abstract machinery. Only as put together in thought, says Rudolf Steiner, is the machine a pure system of centric forces. What Nature provides for it in the way of copper, iron, wood, stone and lubricating oil – what in its solidarity and strength, physically, is not to be arrived at by any process of human reasoning but found out empirically –, therein universal forces are also contained. From this point of view of the inner countereffects, it seems to me to be justified, therefore, to treat mechanical forces in ones thought-picture as universal forces. Spoken concretely, this means: We represent them as vectors in counterspace.

But that is in other words: planewise vectors. "Vector" means: directed quantity. Direction is always linewise. The straight line, however, has two aspects: It is the bearer of a potential movement of points, but also one of planes. The pointwise aspect yields the well-known vectors of positive, Euclidean space. (With all distinctions between so-called free, line-bound (-flüchtig), polar, axial, etc. vectors one still has to do with the pictorial idea of a line of points — though the latter may be, as with "free vectors", displaceable sideways — along which one carries off a certain pointwise length.) Regarding the vector-line of a planewise vector, one will look rather at the negative-spatial distance of two planes borne by this line. N.b.: here, too, the given line bears two kinds of oppositely directed vectors. Let P_1, P_2 be two points of line ℓ; then both P_1P_2 and P_2P_1 represent certain pointwise vectors, where $P_2P_1 = -P_1P_2$. The same holds for planewise vectors.

Now by the "distance" between two planes, however, we of course do not mean the trivial angle between them; for that would be in general a Euclidean-spatial concept. Intended is the negative-spatial distance. In order to determine this, however, we must first determine the "germinating-point" of the counterspace which acts as the "infinitely-distant within". For this, in mechanics, we may begin by taking the mass-point upon which the forces work. We shall only be speaking, therefore, in our first elementary approach, of such force-systems as converge toward a definite midpoint or diverge from one.

With physical-spatial (pointwise) vectors, the infinitely-distant is already given; it is the "infinitely-distant" plane of Euclidean space. If one does not think projectively, one operates with it unconsciously. (I am naturally excepting here kinematics and mechanics in non-Euclidean spaces as they were developed by Lindemann, A.N. Whitehead, and such.) This infinitely-distant plane lies at the foundation of all vectorial measurement. What elementary kinematics takes for granted as step-measure along a straight line, upon which the criterion of a "uniform" motion is based, receives its determination by virtue of the well-known harmonic quadrangle construction from the infinitely-distant point of the line, i.e. from point $\ell\omega$, if ℓ is the line. (The symbol ω will be used henceforth always to mean the infinitely-distant plane of space, likewise O for the infinity-point of the counterspace in question.) Let P_o, P_1, P_2 be three points of the line. We say: The vector P_oP_2 is a times as big as P_oP_1 — whereby a becomes negative when P_1 lies "between" P_o and P_2 — so that we have projectively: the anharmonic ratio $(P_2, P_1; P_o, \ell\omega)$ equals a. — We must be clear about these relationships if we wish to carry over the vector-idea into negative space. If ℓ is the bearer of planewise vectors, then measurement-determinations will involve the inwardly infinitely-distant plane ℓO.

Now in the customary Euclidean way in which vectors are handled, a certain unique starting- or end-point is often involved. E.g., with the Cartesian axis cross, one treats the directed length OP leading from the origin O to a given point P(X,Y,Z) as a vector with components X,Y,Z. Or in kinematics: If a point M partakes simultaneously of different velocities, one represents them as corresponding vectors, all originating from M, and applies repeatedly the parallelogram of motion to find the resulting velocity.

With negative-spatial vectors even more so, a starting-plane may be considered to be given out of the nature of things. It is the infinitely-distant plane of physical (positive-Euclidean) world-space. As I have often indicated in other writings, this plane represents for ethereal processes frequently the archetypal source, the original home, as it were their innermost part. (One must only, with that last phrase, already have changed one's spatial feeling around; therefore I don't normally use it, to avoid ambiguities.)

It is not necessary that plane ω be taken as starting- or end-plane; otherwise there would be in counterspace no such things as "free" or "line-bound" vectors. But it often presents itself naturally. This will be especially the case with physical processes because – as we know from general anthroposophy – the inorganic has its body of formative forces in the widths of space and only becomes an organism when grasped as entire cosmos. Here, however, the physically-spatial cosmos is particularly intended, for whose archetypal determination the "infinitely-distant plane" ω is the most essential one. – In what concrete way the plane ω proves to be the starting- or else end-plane for mechanical force-vectors – how intimately this thought-picture agrees with the known facts and appearances – will soon become evident.

2. Force-Trihedra and -Tetrahedra in Counterspace.

Let us take now the simplest example: When do three forces, that come to bear upon a body, hold one another in balance? The first necessary condition contains already in its form the archetypal polarity of space. The lines of force must meet both in a point and in a plane. I.e. they are all three directed toward a single point of the body, say its centre of gravity. We shall want to have a look at this first as "mass-point" on which the forces work, and choose it as "germinating-point" O of its counterspace.

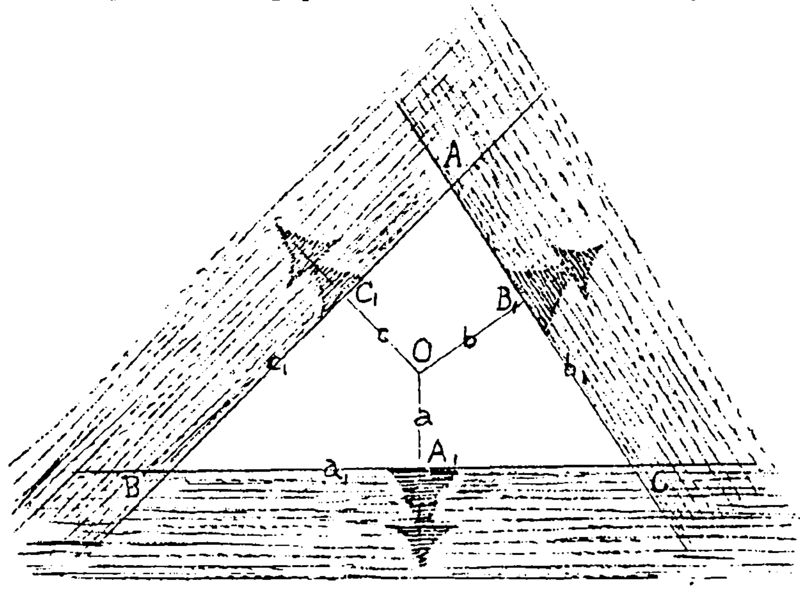

Figure 1

We represent the forces now as planewise vectors and take ω – the infinitely-distant plane of physical space – as starting-plane for all three. As vector lines we take those lines in ω that lie perpendicular to the lines a,b,c in Figure 1 on which the forces work [n.b. not touching, but skew perpendicular, as those lines lying in ω forming respective celestial equators to a,b,c as polar axes]. The respective plane [α, β, or γ – not shown] moving inward from ω parallel to itself and indicating by its end-position – a_1, b_1, or c_1 [in right section] – the relative measure of the force in question, represents as "counterspatial vector" the direction and intensity of that force. The three planes form a prismatic trihedron, borne by the infinitely-distant point, which stands at right angles to plane abc. The farther a particular plane comes in toward O, the greater the force represented. In fact the negative-spatial distances inward – ωa_1, ωb_1, ωc_1 – are proportional to the reciprocals of the vertical distances of the three planes from O in positive space.

Now the triangle $a_1 b_1 c_1$ which the above trihedron draws in plane abc, however, has the same form as the conventional force-triangle from which one would normally read off from the given angles the relative sizes of the forces holding one another in balance. One only needs to turn it by 90°. Thus we know by experience (n.b. it is a matter of experience, not something which can be arrived at by thinking alone, — cf. Rudolf Steiner on this!) that the intensities of the forces must be related to one another as the Euclidean lengths $BC : CA : AB$. — It is then immediately to be seen by elementary geometry that these lengths are related as the reciprocals of the vertical distances $OA_1 : OB_1 : OC_1$ if and only if O is in the so-called "centroid" of the triangle. For we have then $BC \cdot OA_1 = CA \cdot OB_1 = AB \cdot OC_1$ equal to 2/3 of its area.

As a necessary and sufficient condition that the three planewise vectors, with their negative-spatial distances from ω in toward O, represent forces held in balance at a mass-point O, we have: that O must lie in the centroid of the resulting triangle.

The "centroid" is, after all, at the same time the "centre of gravity" of the triangle, but that is mentioned only incidentally, for this triangle is nothing extensive, ponderable; anything but! It should far rather be remembered that "centroid" is a purely mathematical concept. From the theorem of Desargues it follows (cf. my Strahlende Weltgestaltung, §181) [or H.S.M. Coxeter, Projective Geometry, §7.31] that every line in the plane of a triangle evokes its "pole", its harmonic reflection inward, with respect to the triangle. The centroid is then the inner reflection of the infinitely distant line. Thus when the counterspatial vector-picture, Figure 1, evokes in this sense a harmony between the mass-point, bearing forces, as infinitude of counterspace and the infinitely-distant plane of space, then it represents truly a balance of forces. Herein, however, is already contained the reinterpretation of the "parallelogram of forces", for this latter is only another form of the conventional force-triangle. If one assumes, for sake of argument, that the counterspatial representation is the original (whether and to what extent this is justified I leave open for the moment), then the Euclidean representation — force-triangle and parallelogram, on which one measures the force-intensities by external lengths — is valid precisely by virtue of the mutual polarity of the spaces. It is quite possible that for centuries one has measured by means of external spatial quantities what, by nature, belongs not to an external but rather an intensive space. For the harmonies of the world can also conceal the nature of things from human intellect in this fashion, by making things all too easy for it!

We take now the most obvious example: the balance condition for four mechanical forces applied to a body. It is no longer necessary that the force lines lie in a point and in a plane; the one or the other may be the case, or they may all be skew. In the latter case they must all four be generators of the same ruled surface (hyperboloid of one sheet, hyperbolic paraboloid), — again a matter of purely projective determination! We shall leave that to one side to start with, however, and think of four forces that meet in a point, — act on a single mass-point. Again, we consider this point O to be an inner "infinitude" of a negative space and represent forces by planewise vectors, all originating from ω and lying along (physically infinitely-distant) lines at right-angles to the directions of the forces. The negative-spatial distances of the end-planes of the vectors — $\omega a_1, \omega b_1$, etc. [where a_1, b_1, etc. are always understood to be right sections of the actual end-planes α_1, β_1, etc. for convenience in 2-dimensional illustration] — inward toward O are to represent by their mutual relationship the relative intensities of the four forces. How is the condition for balance now revealed in this negative-spatial picture? The four end-planes form a tetrahedron surrounding the point O. Again, the necessary and sufficient condition must read: that this point must come to lie in the centroid of the resulting tetrahedron. Here, too, "centroid" is a matter of purely geometric determination. By virtue of the 3-dimensional theorem of Desargues, a tetrahedron evokes for every plane of space its pole, which is like its mirror image inward. The centroid is the point that results in this way from the infinitely-distant plane of physical space. Thus, if and only if ω and O stand in this mutually polar, harmonic relationship to one another with respect to the force-tetrahedron, is there a balance of forces. — The proof is in this case not quite so easy to give, but may be found without too much difficulty. Euclideanly interpreted: Draw an arbitrary tetrahedron whose planes stand at right angles to the force-lines. The forces are in balance when their intensities are proportional to the areas of the respective triangular faces of the tetrahedron. It is a theorem of "graphic statics".

George Adams

THE LEMNISCATORY RULED SURFACES IN SPACE AND COUNTERSPACE

Memorandum

on the lemniscatory ruled surface and its plane and twisted curves (lemniscates, circles, conic sections, planetary loop-curves, spiral lemniscates, etc.).

London,
May 1939.

Dear Dr. Vreede,

The lemniscate-model starts, if I may characterise it first from the purely geometrical side, from two basic thoughts. On the one hand it is the way in which the lemniscate itself arises out of the interplay of positive and negative spaces. If in the plane picture, i.e. in two dimensions, in ordinary Euclidean space, one takes a centre (O, Figure 1) and lets concentric circles grow out from it, then as the circles become ever larger they tend toward the infinitely distant line. If, over against that, one now thinks of a negative space in the same plane, then there belongs to it an, in first instance, unique point, just as there belongs to Euclidean space something unique of peripheral nature, namely in this case the infinitely distant line. It is the ur- or germinating point of the ethereal space (U, Figure 1).

In order to determine fully the Euclidean plane, however, we need not only the infinitely distant line, but also within it the circling pairing (elliptic involution) of point-pairs lying at right angles, i.e. of those point-pairs which are polar conjugates with respect to every possible circle in that plane. (If we omit this, then we are left with an "affine" space within which we should have no criterion for distinguishing between circles and ellipses.) We think of this involution as having to do with two "imaginary points", one of which is connected with the circling motion from right to left – anti-clockwise, – the other with that from left to right – clockwise. For if we follow the point-pairs of the involution one after another in continuous succession, we must always do this in a circling motion which we may carry out in two senses. The twofoldness of the senses corresponds to the analytical twofoldness of the imaginary roots of a quadratic equation, e.g. when seeking analytically for the common points of a conic section with a line passing by it, outside – that is, in this case, of a circle with the infinitely distant line. (N.b.: I write, as is commonly done in modern mathematics, "imaginary" instead of "complex", calling thus also the general complex number $x + iy$ "imaginary".)

The two infinitely distant imaginary points of which we are speaking here are called the two "circling points" of the plane.

Analogously, for the ethereal space, which is to have its ur- or germinating point in U, to receive the unique determination befitting its form – that is to say, to fix it with sufficient certainty in its relationship to the ordinary, Euclideanly rigidified space within which we are moving with our drawings, models, and mental pictures – we must think of that ethereal space with the ur-point U as having a circling pairing of lines, an "imaginary line-pair", given once-and-for-all. We are quite free in the formal choice of this circling line-motion, able to think of it also as elliptical to any desired degree, with principal axis-pair (Figure 2) lying in any desired manner to the already given direction UO, resulting in interestingly metamorphosed lemniscates and Cassinoids. But we shall assume in all that follows the most natural thing: the circling line-motion of the ethereal space in the ur-point U is to be in perspective with the corresponding point-motion of the physical space in its ur-line (the infinitely distant line). In other words, we imagine in U the circling motion of a right-angled line-pair, again in two senses – clockwise and anti-clockwise (Figure 3).

In the physical space we thought of concentrically growing circles. What corresponds to that in ethereal space? In order to obtain concentric circles we had to make an initially free choice of mid-point (O). So, for ethereal space, we should have to choose freely some "mid-line". From this, there would move, inward, peripherally-woven curves (conic sections) tending toward the ur-point U, reaching in it their "inward infinitude". They should come closer in their form to the once-and-for-all given imaginary line-pair of this ur-point – that is, to the orthogonally circling line-pairing in this point U, – just as the circles of the physical space do to the pair of imaginary circling points in the line at infinity. That, however, means that they must be conic sections which have in the ur-point U a common focal point. That the chosen "mid-line" (o) belongs to all these conic sections, however, means that this line is the polar of the ur-point U for all the conic sections, i.e. the directrix line corresponding to the common focal point. "Mid-point", after all, in the physical space means pole of the infinitely distant line w.r.t. the curve in question; likewise "mid-line" in the ethereal space means polar of the ur-point. Thus we obtain as counterpicture to the family of concentric circles, a family of conic sections with common focal point and common directrix line (Figure 4). The interplay of such a family with the family of concentric circles would – according to the varying position of the mid-line o in relation to the previously chosen points O, U – lead us again to metamorphosed lemniscates and Cassini curves.

Now, however, we shall make a still further, simplifying assumption. We shall let, namely, the ethereal mid-line coincide with the ur-line of the physical space, i.e. with the infinitely distant line u. Nota bene! we abandon thereby the full application of the "duality-principle", for according to that, if we set o = u, we should have to let the physical mid-point, i.e. O, coincide with the ur-point of the ethereal space: O = U. In the formation of the Cassini curves, therefore, the central and the peripheral aspects no longer play the same roles. One does, by the idea of it, have to do with two essences each, central and peripheral: with u and U as once-and-for-all given ur-line and ur-point of the two spaces; with O and o as the chosen central and peripheral "middles" of the two curve-families. Yet we allow o with u, not however O with U, to coincide. If one wishes to think it through consistently, according to the duality-principle, then the formation of the family of Cassini curves, including the lemniscate, demands a polar curve-formation, which may sometime perhaps merit our attention. But we remark this here only as an aside, for the sake of full clarity. Cosmically conceived, the separation of the points U and O, and with that the whole way in which the lemniscatory forms arise, could be said to correspond to the separation of Sun and Earth (or of Sun and Moon-Earth). To begin with, we know only one kind of peripheral essence, namely the infinitely distant plane of our physical space. A multiplicity of concrete centres (namely the centres of the heavenly bodies) is familiar to us. Would one then be dealing, in ethereal view, with a multiplicity of ethereal middles, i.e. of particular planes in cosmic space? Only then would the polar reversal of the lemniscate-formation receive a concrete meaning....

We take now the natural measures of the physical resp. ethereal space and allow, according to them, the physical circles to grow outward for a while, but then the ethereal circles to grow inward. (The ethereal family of conic sections becomes, after all, a family of concentric circles when the common directrix or mid-line moves away to infinite distance, whereby the ur- or focal point U, pictorially speaking, becomes their mid-point, while still retaining — according to ethereal space — its wholly different function, that of the "inward infinitude".)

Measures are always rhythms, rhythmically repeating processes. These processes have their origin in projective space, neither one-sidedly physical nor ethereal (cf. my fundamental essay in the journal Natura, "Physical and Ethereal Spaces"). Its idea is therefore common to both spaces; so it makes sense to relate the rhythmic steps of the two spaces to one another.

Projective geometry admits, to begin with, three kinds of elementary measurement-rhythms. I call them in English quite simply growth-measure, step-measure and circling measure, whereby the word "measure" retains in a beautiful way its two meanings, that of a geometrical unit-amount, or rhythm (verse-metre, dance-movement, etc.). In scholarly language they are called hyperbolic, parabolic and elliptic. But it is time that more popular and meaningful names were chosen.

These measures depend upon repeated projective processes — projectivities, mappings in one dimension, under which in each case certain elements remain at rest. The growth-measure is based upon two real elements ("hyperbolic" because the hyperbola contains two special real elements, the two infinitely distant points with the asymptotes tangent to them). Under a step-measure these two resting elements become fused into one ("parabolic" because the parabola contains only one special element, namely the one infinitely distant point along its axis). The circling measure is based again upon two, this time conjugate imaginary resting elements ("elliptic" because the ellipse has two conjugate imaginary points in common with the infinitely distant ur-line of our space).

As soon as one brings these purely projective processes into the given physical space of our day-to-day inner mode of vision in such a way that one takes as the resting elements certain ur-elements of this space, then they gain the visual forms with which we are familiar, in which they — seen from a certain standpoint — display their archetypal natures, and from which we take their names.

If under a step-measure, e.g. along a line, we set the resting point of the process as infinitely distant point of the line, then there arises the picture of equal-sized steps, as they would be paced off by any being moving about, in first instance, freely in physical space. If, however, we take the resting point to be finite, then we obtain a perspective picture of such a regular stepping-motion, whereby the resting point appears at the same time as vanishing point and, as the process demands, assumes the role, visually, of a two-sided accumulation-point.

If under a circling measure, e.g. among the lines of a point in a plane, we base the process upon the two imaginary circling points in the infinitely distant line of the plane — with the imaginary lines which arise from them perspectively in the given point (Figure 5) — as resting elements, then we obtain the picture of a regular, equi-angular circling motion. All other in the true sense "circling", circularly regular processes may be lead back projectively to this archetypal ur-picture; so, among others, the circling motion of points along a line through the infinite distance and beyond, returning back from the other side to the starting point (Figure 6 as example).

And if, finally, under a growth measure we displace one of the two resting points out into the infinite distance, then the other appears over against the first, with a certain inner necessity, as mid-point (Figure 7). Since, however, the process is based on the two resting points in thoroughly equivalent manner, this has as a consequence that this mid-point reveals itself to be the peer of the infinitely distant point. It is just in the growth process that there are always two infinitudes involved. Here, in the naturally-given archetype of such a process where we actually place one of these two out into the infinite distance of our space, into the outermost part, the other one steps over against it into the innermost part. The growth process, by its very nature, runs its course between two such poles. — As the step-measure is apparent in the animal and human realms (volitional motion, in a line), so the growth-measure is apparent above all in the plant-world, whence the most beautiful pictures of the latter are to be found rather in the realm of the two- than one-dimensional. It is just the picture of concentrically growing circles which shows the natural archetype. Here the two resting portions of the process, instead of being two points, are the mid-point and the infinitely distant line of the plane. Since, however, every radius going out from the mid-point meets this line in an infinitely distant point, we find the growth-measure mapped, depicted along each radius once again in the typical one-dimensional form of this measure (viz. Figures 7 and 8).

Growth- and circling measure on either side differ from the step-measure transition-case between them in that there is essentially only one kind of the latter, to which all others are in a sense equivalent (any given step-measure may be readily projected into any other), whereas the former harbour an infinite multiplicity of individual forms. Any given growth-measure is based upon a definite real number, the so-called ratio-number, which is literally the base of successive stages of growth as a power-series, indicating in what ratio the given form (e.g. the circle, Figure 8, measured by its radius) grows at each stage. In Figure 7, it is the cross-ratio of successive points being sent into one another by the process with respect to the resting pair. To be more precise, there is a reciprocal pair of numbers involved. Namely, in that the process runs its course between two infinitudes which are peers of one another, it is in each case the same individual process which corresponds from the one point of view to the definite number x and from the other to the inverse number $1/x$. In Figure 8, for example, as one easily sees from the in- and circumscribed squares, these are the numbers $\sqrt{2}$ and $1/\sqrt{2}$; indeed, this is true in two senses. If we think of the picture in positive space (infinitude in the exterior, null-point in the interior), then $\sqrt{2}$ gives the ratio of outward growth, $1/\sqrt{2}$ the ratio of diminishing circle-radii inward. If, instead, we think of it in the sense of negative space (ur-point as infinitude in the interior, null-sphere in the exterior), then $\sqrt{2}$ must be considered as measure of growth inward, $1/\sqrt{2}$ as measure of diminishing toward the periphery.

In the circling process we have the pure number as measure of angle (as a fractional part of π). This number, too, has to do with formative processes which reveal themselves in polygons. As is well known, it is connected (logarithmically) also with the cross-ratio of the projective transformation, only that here the latter is imaginary and the cross-ratio itself refers to the pair of imaginary resting elements. The two inverse numbers x and $1/x$ appear in the logarithmic form of the angle-measure as + and − ; if one considers the angle as positive in the one, then negative in the opposite sense of rotation.

In step-measure such a pure number is completely absent; thus one can never indicate the size of a step-measure as pure number but only as a named number. In so doing, one appeals again to the ratio-number of growth-measure (ratio to measurement-unit).

Step-measure is the transition-case between the two others in that e.g. two real points – basis of growth-measure – fuse into one; (hereby step-measure arises) – and from thence, though beyond the pale of the visible as it were, re-emerge as twofoldness of points, namely as imaginary points – basis of circling measure.

If we now let the physical circles, Figure 1, go outward, beginning from point O, and at the same time the ethereal circles go inward toward U, beginning from the periphery ($o = u$), while relating the measures of the two rhythms to one another in the simplest way, there arises the lemniscate. This simplest relationship results if we let those O- and U-circles which touch one another in mid-point A correspond to one another as being simultaneous. The intersection-points of corresponding circles then yield the lemniscate, whether we take step- or growth-measure as our basis (Figures 9 and 10). The other Cassini curves, however, arise only under a growth-measure, and do so according to the following criterion. In whatever fashion we may relate the inward and outward growing circles to one another, there will always occur one moment when those growing inward and those growing outward yield, just for an instant, a circle of the same size at the same time. If these same-sized, simultaneous circles are larger than those touching at A (Figure 11), then there result simple, unicursal ovals; if on the contrary they are smaller (Figure 12), double ovals.

Let the radius from U, or from O, to the mid-point A be equal to 1. Let the radius of the same-sized U- and O-circles corresponding to one another be a. For $a = 1$ we have the lemniscate; for $a > 1$ the simple ovals or "biscuit"-shaped curves; for $a < 1$ the double ovals. Let us assume that in one unit of time the physical circles grow – under a growth-measure, therefore exponentially – by the ratio ξ of their radius at any given moment. Then their radius at time t is equal to $a\xi^t$, if we reckon the time from the moment of same-sized circles; i.e. negative time if the physical circles have not yet reached this size.

The ethereal measures then project themselves into the physical space. For the ethereal radii are to be conceived, according to Rudolf Steiner's "Astronomy Course" (Das Verhältnis d. verschied. Naturwiss. Gebiete z. Astronomie. XVth Lecture, Figures 6, 6a), as coming in peripherally from the outside. But what is left over by an ethereal circle as its complement inward as far as the ur-point U has a physical measure which corresponds reciprocally to its measure in ethereal space. (This simple relationship holds because we assumed the null-sphere of the ethereal to coincide with the infinitely distant line of the physical space.) Thus if R_u is the actual, ethereally-peripheral (spherical, according to Dr. Steiner's way of expressing it and as I believe I understand him) "Radius" of an ethereal circle, and r_u the corresponding physically-apparent radius, then we set $r_u = 1/R_u$. Now let r_o be the radius of the corresponding physical circle with mid-point O. We take our departure from the thought that the ratio of the ethereal measure of an ethereal circle to the physical measure of a physical circle remains constant, by virtue of the ethereal circles growing from the outside inward, the physical ones from the inside outward. That is, $R_u : r_o$ = constant. For the same-sized circles corresponding to one another we set $r_o = r_u = a$, that is $R_u = 1/a$. Consequently $r_o : R_u = a^2$. We have thus determined the measure of the constant: a^2 in the physical, $1/a^2$ in the ethereal space. Corresponding to $r_o = a\xi^t$ we have henceforth $R_u = (1/a^2)a\xi^t = \xi^t/a$. And according to this, the constant product $r_o r_u = a^2$ results as the fundamental equation of the Cassini curves out of the simultaneous inter-growth of physical and ethereal spaces.

Under a step-measure we obtain for one drawing, only, the lemniscate; the other intersection-points of the O- and U-circles result no longer in Cassini but more complicated curves. We set therefore $a = 1$ and take our departure from the circles touching in the middle (Figure 9). We now divide the radius $OA = 1$ into a certain number of parts, let us say n. (In Figure 9 we would have $n = 4$.) This time we set $t = 0$ for the beginning of the process, that is we reckon the time from the moment the physical circles begin from the mid-point O and the ethereal ones from the periphery $u = o$. (N.b. — Under a growth-measure we could not do this, for potentizing, exponentiating, has neither beginning nor end.) We assume that the physical circles grow by one part, i.e. $1/n$, in each unit of time. Thus $r_o = t/n$. (The unit-circles touching one another are reached when $t = n$.) Now for the ethereal radius we have again that $R_u : r_o$ = constant; but this time the constant is 1, for we now have $a = 1$. Thus $R_u = r_o = t/n$; $r_u = 1/R_u = n/t$; $r_o r_u = (t/n)(n/t) = 1$.

The interesting thing about step-measure (Figure 9) is that the lower portion of the lemniscate (in the region of the metabolic system, comparatively speaking, — preponderance of radial space) arises more quickly than the upper portion (head-system, — preponderance of spherical space). The radii of the physical circles yield an arithmetic series; the physically-apparent radii of the ethereal circles an harmonic one ($r_u = n$, $n/2$, $n/3$, $n/4$, etc.).

The important thing is of course not these analytical data; these serve only to confirm and check the logicalness of the thought which we grasp more intuitively, visually. One must try to picture to oneself inwardly the moving process: the physical circles beginning from O and growing outward, simultaneously the ethereal ones from the infinite distance $u = o$ weaving inward; the interpenetration of the circles beginning below O, sweeping around O, attenuating accordingly in the middle; then, while the physical circles grow ever further and the ethereal ones come in towards U, the places of penetration closing in around U, until above U the process leaves the realm of the visible, as the physical circles become all too large and the ethereal ones all too "small"...

You know the interesting colour relationships, wherein one naturally takes reddish-yellow circles for the physical ones beginning from the mid-point, with maximal intensity there where they begin; blue-violet for the ethereal ones, with maximal intensity again at the place where they begin, which for them is the infinite periphery. Thus might we link the purely geometrical, qualitative thought to the inner truth of colour-polarities.

One can and should, of course, also reverse the process by thinking instead of a growth, a shrinking of the physical and ethereal circles. The physical circles grow smaller from the infinite periphery toward the mid-point O; the ethereal ones, literally speaking, do the same from their infinitude within (U) toward their place of origin, the world-periphery. As they pass one another, the two kinds of circles again draw a lemniscate resp. Cassini curves in space; this time, however, beginning above U and finishing below O.

The lemniscate arises out of the idea of interpenetrating physical and ethereal circles in another way, as well, namely in the sense of Figure 13, by creating a projective relationship between two families of circles tangent to two lines at right angles through a mid-point A.

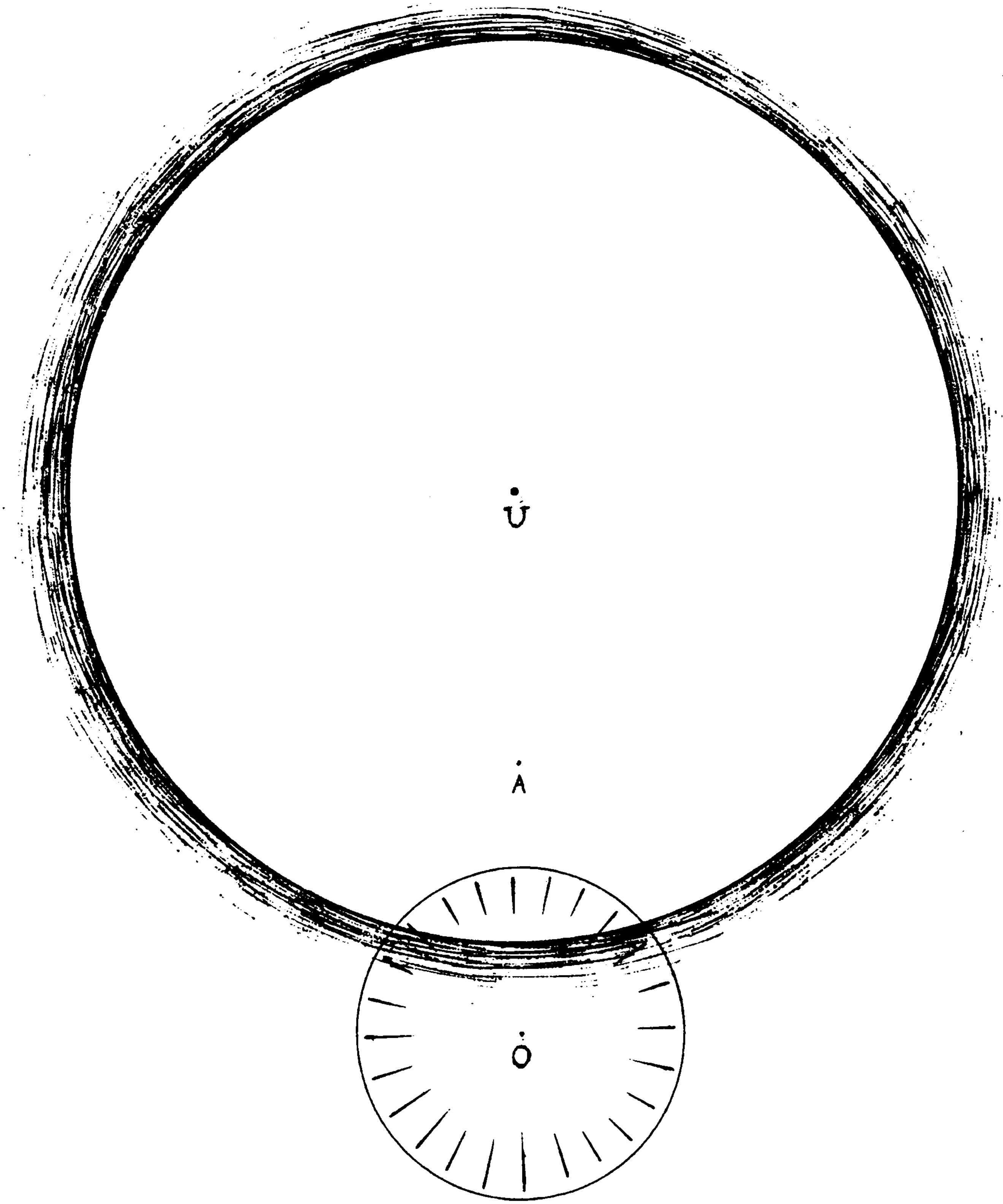

Figure 1

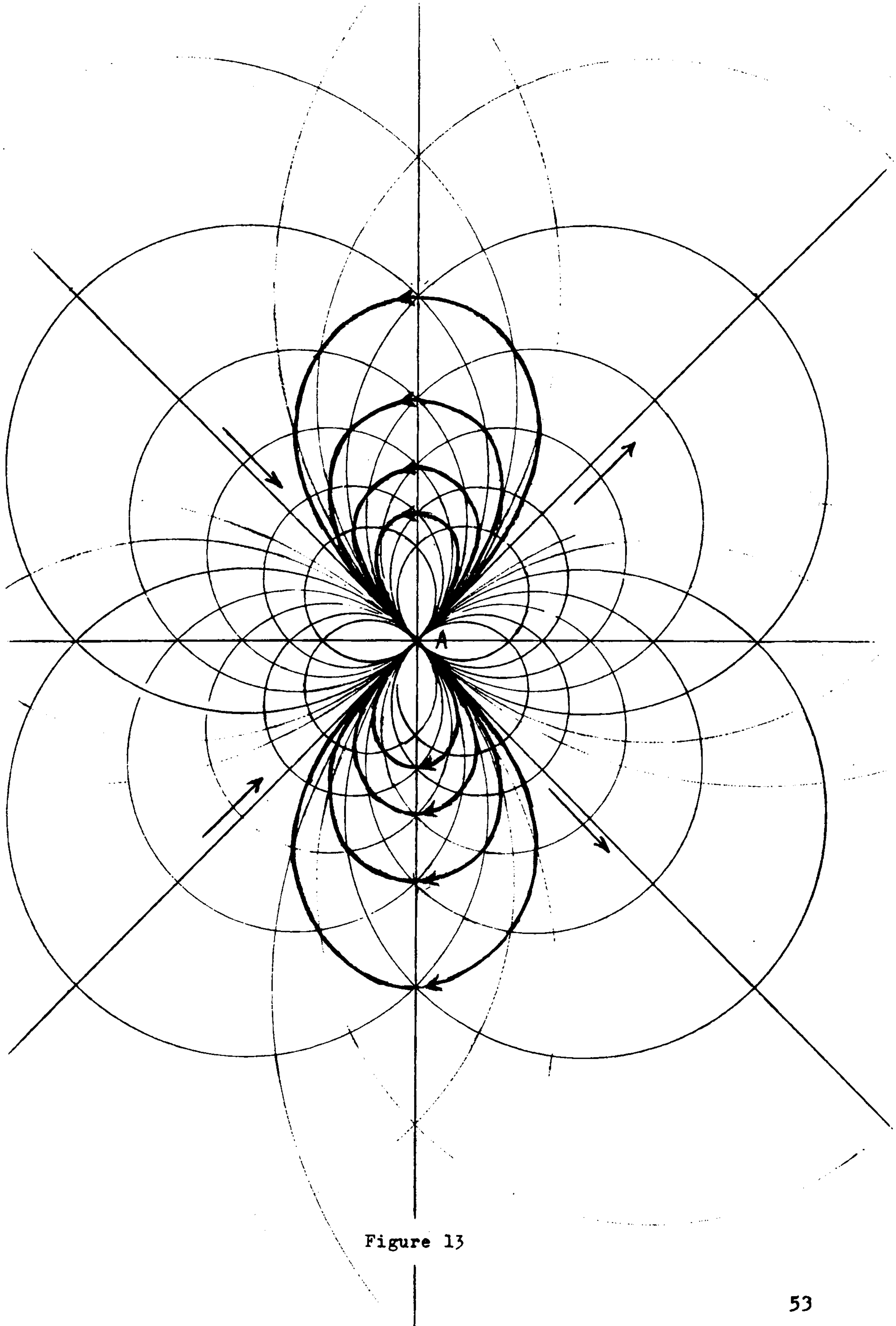

Figure 13

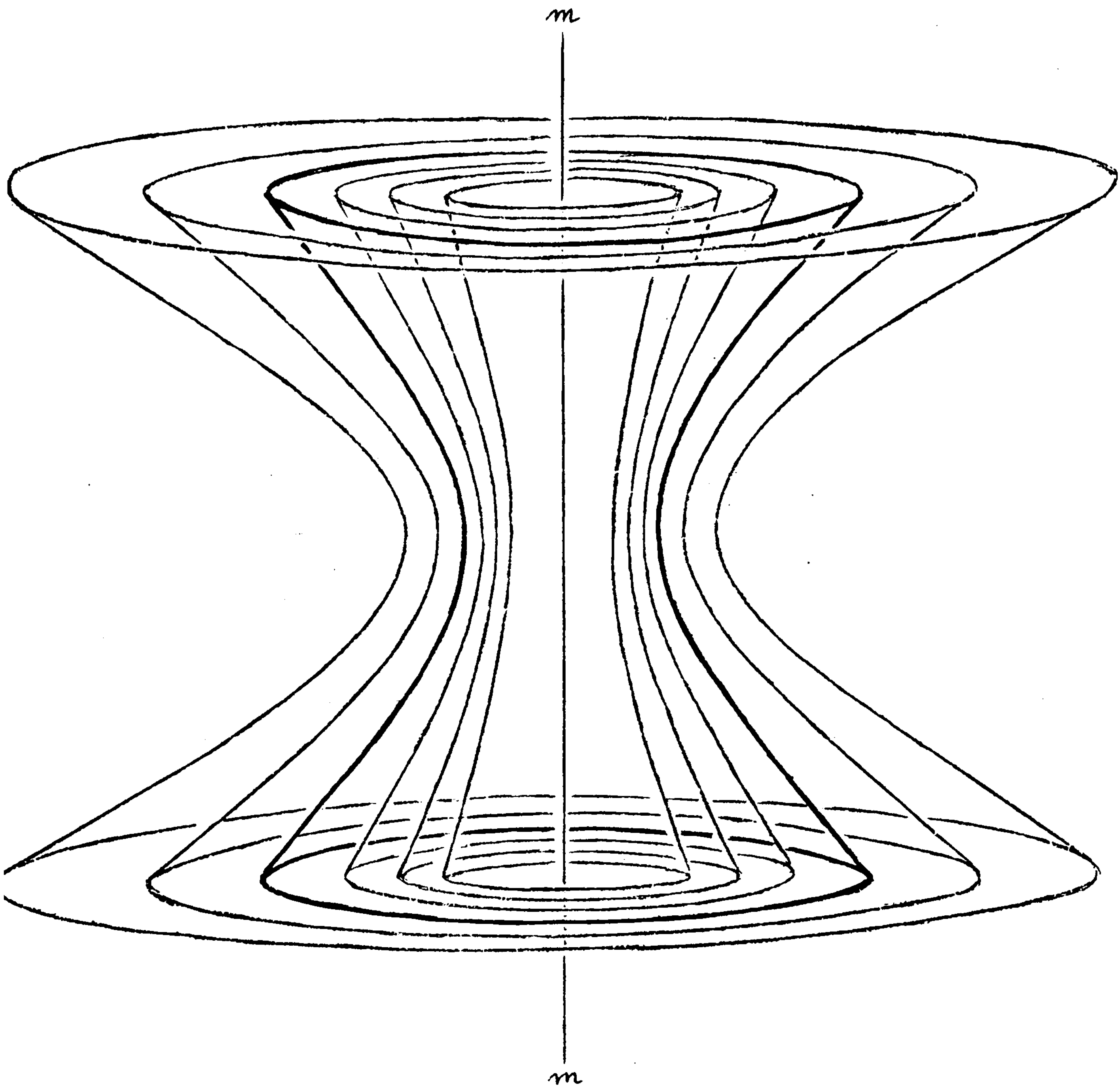

Figure 26

The <u>initially</u> intended hyperboloid (page 16) is drawn somewhat more strongly.

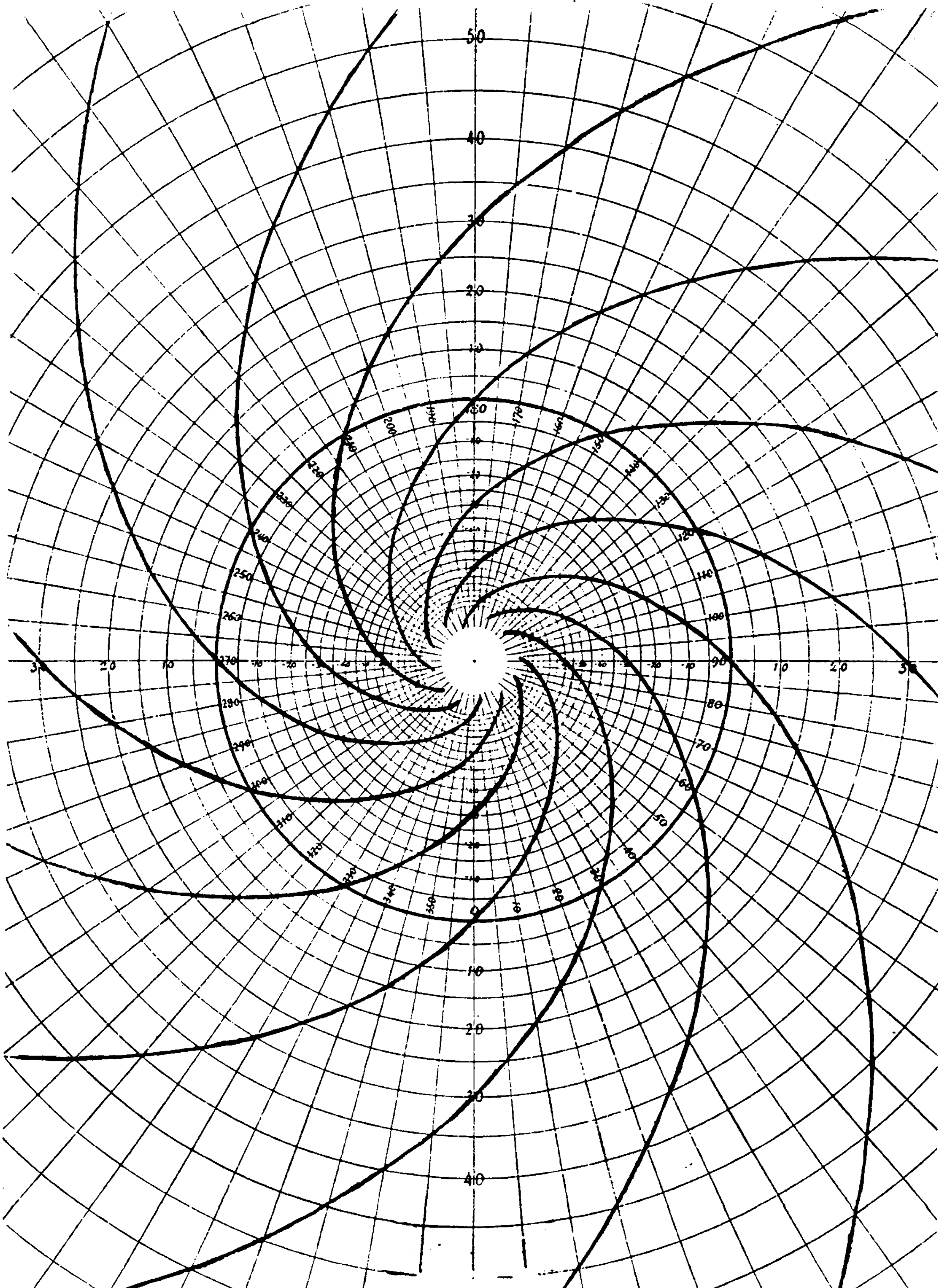
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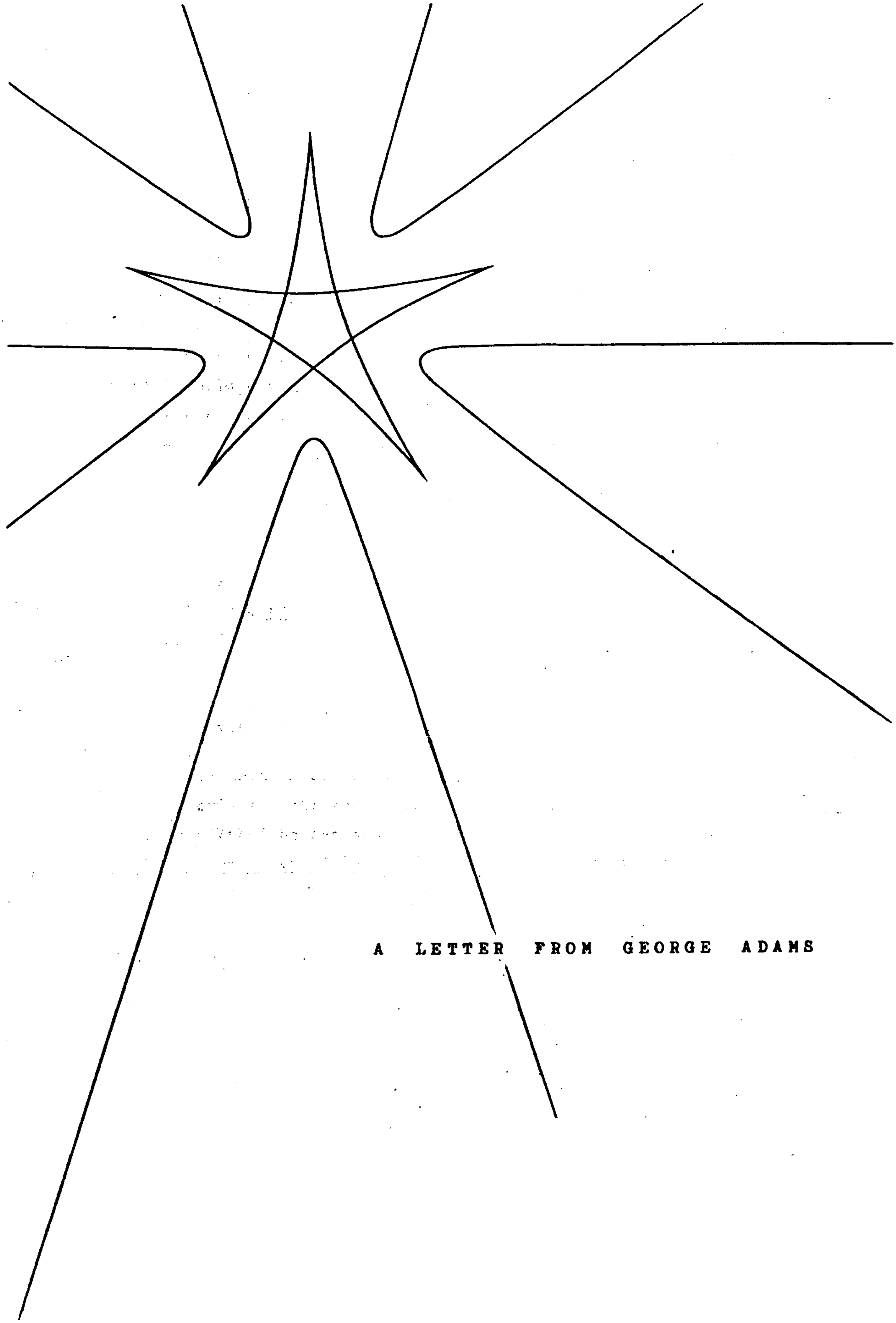
Figure 42



A LETTER FROM GEORGE ADAMS

22 Melcombe Street
London N. W. 1

9th Dec. 1942

Dear Olive,

Today I want to tell you a little more about the present stage of my work, as I promised to do. It has brought me unexpectedly near again to what I was working at about 25 years ago, and it will therefore be natural for me to put it in a wider setting, which I think you will also find helpful. I shall be telling something of my own scientific striving, reaching back to the time even before I went to Cambridge, full of unlimited hopes and ambitions; also of the contacts I had with others and the way I have been guided. At the same time you will get a little picture of what is working in the Spirit of the Time — both for and against, as it were.

You know what Dr Steiner has told us about the present, scientific age, especially in connection with the influence of Francis Bacon, and the stream of Arabism, and how all this stands over against the Michael being. I shall be reminding you especially of what he said in this connection, about "Nominalism and Realism", — for instance in the last but two of the letters he wrote us, just before his death ("Das scheinbare Erlöschen der Geist-Erkenntnis in der Neuzeit", March 1925)[1].

Our individual destinies are interwoven with the destinies of the time in which we live. I can remember, as a very little boy, overhearing someone speak about "Naturforscher"; I pricked up my ears and decided in my mind: That is what I want to be. Embarking on the study of Science with this feeling that one wanted to understand and penetrate more deeply into NATURE, one little knew what one was in for! I was about 15 when I changed from the classical to the scientific side at school and took up chemistry with great enthusiasm, though a little bewildered by the scholastic system of scientific theory.

There is, or was at that time, very little philosophic background to the teaching in an English school. The first time I became enthused for wider ideal problems of knowledge was about two years later, when my father, having read a good review of it in his Vienna paper, gave me a book that had just been published, by the German physical chemist, Wilhelm Ostwald. It was called, after a saying of Goethe's, "Die Forderung des Tages". I knew enough physics and chemistry by then to appreciate it; also I loved Ostwald because his text-books were so wonderfully clear and vivid. It was this book, really, which first set me going, and as you will presently see it was not a bad direction to start in. That was about Christmas 1910.

Ostwald was one of three men who founded a new direction, one might almost say a new branch of science, — Physical Chemistry. The other two were the Dutchman, J.H. van't Hoff, and the Swede, Svante Arrhenius. Of the three, Ostwald was the most erratic; though Arrhenius too — whose name you will often come across in Dr Steiner's lectures — was one of those scientists who ventured abroad, like children playing all unconscious in a room full of delicate instruments, in realms of philosophy and deeper cosmic

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1. "The Apparent Extinction of Spirit-Knowledge in Modern Times", Anthroposophical Leading Thoughts — The Michael Mystery, London, 1973.

problems. In an amazing degree, Ostwald had the faults — and virtues — of a naive German. Born in the Baltic provinces of Russia, there was happily something a little looser in his constitution, which saved him from the extremes of pedantry. But he was so naive that in the last war — already a famous man, a Nobel Prize winner and all the rest — when he lectured in Sweden on behalf of the German cause, he said such extravagant things that the Germans had to disown him, which they conveniently did by pointing out that he was born in Russia.

He was a marvelous teacher and must have had unbounded energy. Long after his retirement, when about 70 years old, he took up the study of colours — not without some intelligent reference to Goethe's work — and with such good effect that you will find the Ostwald system of colour-designation widely used today in industry and art.

Physics, since the 17th century, had been a science largely permeated with mathematical treatment; Chemistry on the other hand had been predominantly experimental, and the thoughts that imbued it — as in the Atomic Theory — were comparatively simple geometrical thought-pictures, plus a certain amount of the simpler physical theory. It was about the 1880's that these men and others began to make the intermediate region between physics and chemistry into a single science, with the result that today Chemistry is as highly mathematical and technical as you could wish, and a physical chemist like our refugee friend, Prof. Jellinek (himself, in his youth, a pupil of Ostwald), must be a highly accomplished mathematician. Nay more, from the side of physical chemistry, mathematics has increasingly taken hold of the biological sciences also.

To explain the direction in which Ostwald was leading, 30 or 40 years ago, I must refer a little to the history of Physics. Very broadly speaking, one can see three great phases in it: (1) the Mechanical, (2) the Thermo-dynamic, and (3) the Electro-magnetic.

The mechanical phase was dominant until the end of the 18th century and received its classical, scholastic form through Newton. Newton put the laws of force and motion, discovered by Galileo, by himself and others, into so self-contained a mathematical form of definitions, axioms and propositions, in his "Principia", that the study of these laws and of the geometrical forms they give rise to became one of the main branches, even of pure mathematics, for a long time to come. At Cambridge for instance, this Newtonian science of Mechanics (or Dynamics and Statics and it was also called) was one of the main items of the Mathematical Tripos and was studied, no doubt, by generations of "wranglers" many of whom hardly ever did or even saw a real experiment; so much so was it transformed into a pure mathematical discipline.

This "mechanical" phase in physical science reached its height at the end of the 18th century, and you may think of such works as were published then — Laplace's "Méchanique céleste", or de la Mettrie's "L'Homme machine" — the titles are indicative. Many French scholars, with their keen intellect and intellectual-aesthetic sense, excelled in developing the Newtonian science.

The other two phases grew up side by side in the 19th century and became merged to some extent towards the end of it. Speaking of these phases I do not so much mean a sequence in time, but different lines of inner development and discovery.

To explain what Thermodynamics is: one of the main principles of Newton's Mechanics is the "conservation of energy". When you once know how to measure energy, you find that in mechanical processes it is never lost; it only changes form. That at least is the classical Newtonian doctrine; how it is related to reality, is another matter.

Actually however, this ideal Newtonian mechanics would only hold good if there were never any friction or any kind of waste; if every machine were ideally lubricated and behaved, so to speak, like a perfect gentleman.

Whenever there is friction, mechanical energy is lost and in its place heat or light or electricity arises. Moreover there are those more inner processes like melting and boiling and crystallizing, or the burning of coal and other chemical activities, where heat arises and is in its turn somehow transformed into mechanical forces and energies.

The rise of the steam engine made first the engineers and then the pure scientists turn their attention to the conditions under which this happens, and there gradually arose a wider science of the transmutations of energy, taking into account not only the mechanical energies of movement but the energies of heat and light and electricity and of the hidden chemical affinities which, for instance, make acid and alkali boil up with excitement when you pour them together.

This became "Thermodynamics". The form in which these discoveries were expressed was largely influenced by the already existing, very powerful thought-forms of classical mechanics. There was a tendency simply to widen the old ideas of mechanical energy and with the help of atomistic theories (as in Tyndall's "Heat as a Mode of Motion", a 19th century classic) to make everything look as mechanical as possible. But there was one inescapable difference.

In Newtonian mechanics everything can go forward as well as backward. It depends so to speak, which way it happens to start. To put it a little wildly, nothing would be essentially changed, in such a world, if God suddenly decided to reverse the clock of Time. But thermodynamic processes always go one way. You can lose energy by friction – lose it in the sense of scattering it as wasted heat – but you cannot collect the frictional heat and turn it back again into mechanical energy, making it work a machine. If you tried to do so you would spend so much energy – not only mental but physical – in the process, that it wouldn't be worth it; moreover the devices you did it with, would in their turn waste still more energy. Thermodynamic science thus makes you realize that in the physical world things always go in one direction with the advance of Time. In a Newtonian mechanical world every pendulum, as it were, swings to as easily as fro; in the more real thermodynamic world things always run down more easily than up, and you must always let three or four horses run downhill in compensation if you want one to run up.

The thermodynamicists at last conceived another quantity besides energy. They named it "entropy" and said that while the energy of the world always remains the same, the entropy would only remain the same if everything ran absolutely smoothly; in every real process the entropy increases, and its increase is a measure of the extent to which useful energy is lost and mere scattered energy takes its place. Some men like Kelvin went on to say: if this goes on everywhere and for ever, the whole world will at long last be one luke-warm mess, where nothing can happen any more. That is what Dr Steiner so often refers to as the "Wärme-Tod".

With these ideas however, scientists entered into things far more deeply than they had done before. They had to think more subtly. Time became more real to them. Moreover, they were entering more into the inner physical and chemical processes. Mechanics always has to do with finished, self-contained objects, pushing and pulling each other about. Polished billiard balls, perfectly elastic, never breaking, are the ideal objects of Newtonian mechanics. There is space all ready-made and peopled with such objects, and time going like a perfect clock for them to swing by. In thermodynamics on the other hand, we study how things come into being and pass away again; how water turns to ice and ice to water; how steam arises and how it condenses again into drops of dew. Moreover though in fact this always happens with "increase of entropy"; by imagining, like a kind of infinitely distant, never-attained vanishing-point, the ideal process in which the entropy was not increased, we get a delicate conception of how the forces are balanced and what it is that holds the balance.

The fundamental idea is almost absurdly simple: there is one way of avoiding friction and that is not to move at all! The slower you move, the less friction there will be. Now when the forces are balanced there will be no movement; hence no friction; hence no waste. Therefore, while every machine works irreversibly and dissipates its energy; if only we could afford the time and let our machines work infinitely slowly, they would waste no energy at all. But if you want a thing to go slow, you must only just upset the balance, and in the last resort – if it's to go infinitely slow – it must be in balance. Thermodynamics thus became the study of the inner balance of physical and chemical processes.

Towards the end of the century this began more and more to affect the minds of chemists, and it was thus that the science of Physical Chemistry arose, with Thermodynamics as its dominant idea. Prof. Donnan, with whom I had a very interesting interview in about 1917, was President of the Chemical Society two or three years ago, and in his presidential address – looking back to the beginning of the century, when he himself worked under Ostwald at Leipzig –, he called it the "thermodynamical era" in chemistry. Quite other leading ideas have since got hold, but it was really this which first caught my enthusiasm, about the time when I was going to Cambridge.

The mathematical ideas embodied in thermodynamics are really very subtle, and at the end of the last century a few men – Ostwald among them – said to themselves: this is surely an advance on the old, far more external and crude, mechanical ideas. Why then should we go on imagining the world to be filled with mechanically behaving entities, such as atoms? Let us give up creating these figments of the mind and turn all our attention to the actual phenomena and the purely mathematical laws and relationships which they reveal when diligently studied. The pure mathematical relations are the thing; why dress them up in a doll's theatre, in a puppet-shew of crude atomistic pictures?

Thus thinking, Ostwald at some scientific congress in the 1890's gave a lecture which he boldly entitled, "Die Ueberwindung des wissenschaftlichen Materialismus". Not long after, Dr Steiner published the last of the five volumes of Goethe's scientific works in the Kürschner edition, in the earlier volumes of which he had inveighed, philosophically, against the dominant atomism, and advocated the Goethean phenomenalism. In the introduction to this last volume, Dr Steiner refers to Ostwald's lecture, and tho' one swallow doesn't make a summer, he welcomes it as an indication that scientists are at last beginning to see light; tho' he goes on to point out that Ostwald, merely replacing the rigid concept of matter by that of energy, does not get much farther.

About 1906 the Chemical Society in London celebrated the centenary of Dalton, founder of the atomic theory such as it is in modern chemistry. They had the misfortune to invite Ostwald as the guest of honour, and he took the occasion – with many compliments and bows, of course – to shew the atomic theory the door; it had done its good work, and a new phase in science was coming. However, other things were on the way, which set the atomic theory up again, and Ostwald himself, before his death, recanted.

For me however, it had the effect, that long before I heard of Dr Steiner I had in my inmost resolves and in my youthful cheek and enthusiasm abandoned the atomistic way of thinking and resolved on a more purely ideal interpretation of the pure phenomena.

It had also another effect, very beneficial so far as I was concerned. The thermodynamic study of chemistry is really fascinating once you enter into it; at one stage, it does give you a feeling of getting nearer the heart of things. But to make anything of it, you must master the thermodynamics, and that involves a good deal of mathematical and even philosophic training (or at least training in clear and subtle thought), as well as a more all-round study of physics. It was from this side that I was driven to study far more mathematics than I should have done without this. By good fortune, my father gave me the Russian physicist's, Chwolson's text-book of Physics, in which it is set forth in a masterly way and in its true historic setting. In later years, I shared with Ehrenfried Pfeiffer his enthusiasm for Chwolson's exposition.

By the strange system of scholarship exams, I worked hard at chemistry and physics and went to Cambridge for a competitive exam of this kind about a year before leaving school; having won a scholarship at Christ's I had a remaining year at school, partly to get up Latin and Greek enough to pass the Cambridge entrance exam, and partly to do what I liked in.

The outcome was that when I went up to Cambridge in Sept. 1912 I found in the first week that the lectures I was attending seemed to be going over ground already familiar to me. All this was partly precocious and illusionary, but as it happens it led me to an important decision which fortunately my tutors did not discourage. I resolved, in the first year, to take the first part of the Maths Tripos, and only then, in my second and third years, to return to chemistry. In this way I got at least a certain amount of regular study in mathematics, – though even that was little enough and I did not do very well in it.

I must now take up the thread at another point and tell you a little of what I called the <u>third</u> phase in the development of Physics: the Electro-magnetic phase. This too began to blossom forth at the turn of the 18th and 19th centuries. Now here we touch upon a deeply interesting thing. What, in all this development I have been describing, is the work of men, the thought and will of human souls, and what is the working-out of great cosmic forces and necessities – the inescapable inspirations of the Time-Spirit?

I ask the question here, because with electro-magnetism something quite unforeseeable came in, so to speak, out of the blue. All other things we have been speaking of – mechanical events, physical processes, heat and cold, boiling and crystallizing, even the chemical comings and goings, had been known to men from of old. Even the Greeks made crude steam-engines, and the alchemists in their way studied chemistry. But the electro-magnetic forces, save for the lodestone and the trifling attraction of light objects by a piece of rubbed amber or sulphur (electron is the Greek for amber, hence the name), were utterly unknown to man since history began. Now at that time, 100, 150 years ago, they really began to be discovered. Of course the discoverers – Volta, Galvani, Ampère, Gauss and the others, and then above all Faraday – worked hard, and had their individual intuitions. Yet it is like something coming in from unknown cosmic depths, when the real power and wisdom of the electro-magnetic forces becomes known; a whole realm of force, no man could ever have suspected. Dr Steiner has indicated that this had to do with the fulfilment of a certain cosmic cycle, a kind of constellation recurring, of which the preceding period reaches far away back into old Lemuria.[2]

What is important is not only that this gave the physicists and inventors, as it were, a new field of phenomena to play in. More than that, it involved the development of altogether new thought-forms – thought-forms undreamt-of in the Newtonian, mechanical science that had prevailed before. While electricity, it is true, has profoundly to do with matter, it is the <u>inner</u> rather than the outer side thereof; therefore, mechanical thought-forms, where ready-made material objects react on one another in a ready-made external space, prove quite inadequate to the understanding of electric and magnetic forces.

Gradually, men had to learn quite new forms of thought, which, though concerned with forces deeply, occultly connected with the material world, are for this very reason a stage more spiritual than the old naive mechanical ideas.

The most characteristic thing about these thought-forms – known nowadays under the general name of the "field-theory" – is, if I put it very briefly, that where something <u>seems</u> materially to be there, one eventually comes to recognize that there is nothing, whereas where nothing seems to be, one recognizes the essential "something". For instance, in Faraday's conception, if two electrified copper globes confront each other across empty space, we come to recognize that as regards the electrical forces, everything stops once you get inside the absolute <u>surface</u> of the globes; whilst on the other hand the entire space between them and around them is filled with "lines of force", varying in intensity from point to point and weaving the most wonderful geometrical patterns. For this electric <u>field of force</u>, the copper globes which, as you think at first, "carry" the electric charge, are just empty holes. Yet on the other hand the whole character of the field of force depends on the shape and distribution of these empty holes – and, so to speak, in what way and how intensely they are empty.

Something quite similar occurs on another level when a wire, as we think at first, "carries" an alternating current. The perpetual alternation of electrical polarity, brought about by the dynamo to which the wire is attached, induces magnetic forces which you may imagine in the shape of circular rings in the empty space around,

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2. Lecture 1, 2nd Oct. 1916, in <u>Innere Entwicklungsimpulse der Menschheit</u> (Bibl. Nr. 171, not transl.); cf. also Ernst Lehrs, <u>Spiritual Science, Electricity and Michael Faraday</u>, London, 1975.

rings through the centres of which the wire is threaded. The circles expand and contract with every alternation. These magnetic forces in their turn, by a kind of resonance, induce electric forces which tend to keep the current alternating, and by a proper arrangement you can make this go on for a long time, like an invisible pendulum swinging to and fro, even without a dynamo connected. That is in fact the principle of Wireless — which is only "wireless" in the sense that the resonance is carried from place to place without wires; in the receiver there will always be wires to translate it again into a palpable form. Here again, all that is essential goes on in the electro-magnetic "field" outside of and around the material objects; where you would think there was nothing, there precisely is "it". Moreover, the "it" is a piece of pure geometry and mathematics, if ever there was one.

Historically, this field-theory — to some extent begun by Gauss in his studies on magnetism — was developed above all by a man who had the gift of imagination to a high degree but was not a mathematician in the ordinary sense, and expressed his thoughts in pictures, rather than in formulae, Michael Faraday; then in the middle and latter half of the century, Clerk Maxwell, who was a mathematical genius, translated Faraday's imaginations of the electro-magnetic field into some of the most wonderful formulae that have ever been conceived by human mind: Maxwell's electro-magnetic equations, and the whole way of thinking, imagining and calculating that goes with them. They are indeed so wonderful, so simple and yet profound, so wisely interlocked, that the Austrian scientist Boltzmann — quoting, I believe, from Faust — wrote as the motto before his lectures on Maxwell's theory: War es ein Gott, der diese Zeichen schrieb?

It was no accident that Maxwell divined and foretold the existence of electro-magnetic (wireless) waves, though he died young and did not discover them by experiment. That was done by a German-Jewish scientist, Heinrich Hertz, who died many decades ago; yet the times are so interwoven that I believe Hertz's widow is living to this day, a refugee and guest of some younger scientists at Cambridge.

Alas, Maxwell also believed himself to have discovered that Light consists of these same electro-magnetic waves, only of very small dimension. One must admit, his proofs and arguments are convincing. But as men do not know how to distinguish original light from its, as it were, materialized counterpart, this has led to the tragic confusion in all modern thoughts about the Cosmos, that we think the whole Universe is made of electricity.

Within the intellectual and social setting of Victorian England and of the 19th century, Faraday and Maxwell were two of the purest, inwardly most Christian men that have ever lived, and this is surely very significant, for the future also.

I have said enough to indicate how the electro-magnetic phase brought something quite new and characteristic into Physics. Yet, different as it was in many ways, it was linked up with what had gone before; and the main link was in the concept of energy. Once you know how to measure the electric and magnetic energies, you find the same kind of metamorphoses here as in the classical mechanics and in thermodynamics. You can perceive the pulsing of energies in an electro-magnetic wave, with the mind's eye, just as you can in a swinging pendulum or a vibrating spring. Moreover when other forms of energy are used to produce the electrical or vice-versa, the one arises, so to speak, in proportion as the other vanishes. Nay more, here too the thermodynamic principle of irreversibility comes in. You can use heat to produce electrical energy, as we do in many of our power stations; but you can far more easily waste electrical energy in producing useless and scattered heat, since every wire that conducts a current gets warmed, however little, as it does so, and to that extent uses up the current, very likely, where you do not want it used. Through the principle of energy, the old Newtonian mechanics, thermodynamics and electromagnetism all became linked together in a single scientific system. This was the stage science had reached towards the end the last century.

Indeed, just because the electromagnetic forces are so impalpable, the idea of "energy" plays an even greater part than it does where you have something more tangible to deal with. When you buy any other thing you get a tangible quantity of pounds or pints. When you buy your electric current, you buy pure energy, and that is what you pay for. There are many aspects of electric forces, and accordingly many units in which these different aspects are measured: ampères, volts, and so on. To decide what you have to pay, they measure the <u>energy</u> you use: kilowatt-hour, or whatever the unit is called, is a unit of energy. Ostwald in his "energetic" heyday — he never minded how crude he was — said to his materialistic brethren: You say energy is a mere mathematical idea, nothing tangible; very well, the surest sign that it is real, is that in electricity you pay for it, just as you pay for 10 lbs. of potatoes!

Now the development of electrical science, like thermodynamics, also had a very near relation to chemistry. For one thing, one of the first known means of making electric currents — from which in turn one gets the magnetic effects — was by making a chemical reaction do the work. That is what happens in every flashlight battery. Instead of putting the chemicals — say zinc and hydrochloric acid — together straightaway, where they get vastly excited and use up the hidden force of their chemical affinity in mere heat and noise, keep them a little apart and let them get at one another slowly; by proper devices you can, as it were, canalize that latent energy and get electric forces from it. Vice-versa, when we electro-plate metallic instruments and in many another process, we use the energy of the electric current to bring about some desired chemical effect, which would not simply happen of its own accord. It is again the principle of hitching a crowd of creatures, cantering downhill for the sheer fun of it, on to a few whom you thereby oblige to go <u>up</u> the opposite hillside.

It was also Faraday who first discovered the exact relation between electric currents and the chemical reactions connected with them. He was indeed, before all else, a chemist. But the connection was already known, and indeed leaps to the mind. You must imagine that in the first 30 years or so of the 19th century, science (except for pure mechanics) was not yet so cut and dried. There was still some excitement and mystery about such forces as the electric and magnetic. There hung about the very words a touch of the occult. Goethe and his friends were always talking about Polarities; and where were polarities more in evidence than on the one hand in electricity with its Positive and Negative, and on the other in chemistry with its Acid and Alkali? It was inescapable, tho' by no means clear, that in some way the two polarities had to do with each other. One of the greatest chemists of that time, the Swede Berzelius, worked very largely on this "electro-chemical" theory.

In the latter half of the century, when thermodynamics had become well established, all three were clearly linked together. Indeed the electro-chemical cell — whether as battery, where chemistry produces electricity, or as electrolytic cell, where the reverse takes place — became one of the clearest and most beautiful examples of the thermodynamic machine. It happened to be one which you could <u>really</u> make go infinitely slowly and then see recorded, on some delicate instrument, the exact voltage that was needed to balance, i.e. to hold it in check. Herein you had a perfect thermodynamic measure of how strong the chemical affinities in question were.

This electro-chemical way of estimating chemical affinities — that is the inner balance of the forces by which things are made and unmade — closely bound up as it was with the rather subtle thoughts of thermodynamics, was one of the things I learned with the greatest enjoyment while at Cambridge. I practised glass-blowing so as to be able to put together for myself the peculiar-shaped glass vessels that were used. I also had to learn more and more of electricity itself.

At that time these things were comparatively new. Today in any text-book of biology or even applied biology (let alone chemistry) you will come across the mysterious letters pH, written just like that, with a small p and a capital H hanging on to it. In any watery solution, notably if the balance of acid and alkaline tendencies comes into question, you have to do with what is called the Hydrogen Ion; there is more of it if the solution is acid, less if it is alkaline. In physiological processes this is most important. pH stands for the inner pressure of the hydrogen ion. It is a conception entirely based on the thermodynamics of electro-chemistry, developed in the classical age of physical chemistry about the end of the last century. The pressure referred to is what they call "osmotic pressure"; you may remember the experiment I was doing with the children two years ago to shew osmotic pressure in solutions, when one of the children got me a sheep's bladder. This "pressure" is so called because it has the power to counteract physical pressure; also because the scientists of that time did not dream there were forces other than of the nature of pressures; but it is really more like a kind of suction — an inner, etheric, chemical force, belonging, I believe, more to the realm of negative than of positive space. Some scientists, like <u>Willard Gibbs</u> in America, spoke of a kind of "chemical potential" and went a long way towards thinking of it in contrast to the forces of gravity. All this was fascinating to me; encouraged by the teachings of Ostwald and others, I felt it was leading on towards a new era in science, and I worked hard, really to master all the ideas which it involved.

Meanwhile, however, other ideas and discoveries were coming to the fore, tending to bring on again with redoubled force the atomistic thought-forms which one hoped were being overcome. Being very biassed, I tended to neglect these other discoveries. I shall tell a little more of them presently, but before doing so, I must look back again through the centuries and refer to the parallel development of mathematics.