A SHORT

# PRACTICAL TREATISE

ON

# SPHERICAL TRIGONOMETRY;

CONTAINING A FEW SIMPLE RULES,

BY WHICH

THE GREAT DIFFICULTIES TO BE ENCOUNTERED BY
THE STUDENT IN THIS BRANCH OF
MATHEMATICS ARE EFFECTUALLY OBVIATED.

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# PREFACE.

It will perhaps be wondered at, after so much has been written on this subject, that I should have the temerity to offer my mite to the notice of the public, or to imagine myself capable of throwing any light upon a branch of Mathematics that has been so much illuminated by the works of the learned. I trust, however, that when the reader has heard my explanations, he will not consider that I have engaged in a useless or unprofitable undertaking. Much indeed has been written by learned and experienced Mathematicians, but their works seem rather addressed to the proficient, than to the uninitiated student in the science. Their long formulæ, complicated rules, and demonstrations, &c., perplex rather than instruct the beginner, who perhaps, terrified at the mass of difficulties before him, gives up in disgust the study of a subject, which, treated in a simple manner, he had easily acquired. The great fault of all elementary treatises on this branch appears to be the crowding too much upon the mind of the students, and distracting their attention with useless rules and demonstrations, which retard rather than assist their progress. This I have found by experience, both in my early studies, and in the extensive practice I have since obtained in communicating knowledge to others; and I have frequently in one hour's conversation enabled a pupil to master a subject which he had in vain attempted to acquire by the perusal of the ordinary rules. In short, I felt that a plain practical Treatise on Spherical Trigonometry, in which the pupil's attention should not be distracted from the subject before him, and in which the rules should be as simple and concise as possible, would much facilitate the acquirement of the subject. I have therefore written in the same manner in which I should have explained it by oral communication with my pupils. The formulæ which I have constructed for solving each Case in Spherical Trigonometry will, I hope, obtain the sanction of the Mathematician for their correctness, and the approbation of the student for the ease and rapidity with which they enable him to master the science.

OLIVER BYRNE.

# INTRODUCTION.

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By the word Sphere is generally understood any circular body; but the term was appropriated by the ancients to an assemblage of circles and constellations representing their "Primum Mobile." The invention of this Sphere is ascribed to various persons, but it is evidently too remote to be traced by any authentic history. The Chinese had a knowledge of the Sphere at a very early period,* from whom it was probably transmitted to the Chaldeans, thence into Egypt and Greece; but it

* "Xuni, 2400 ans avant J. C. fit faire une sphère d'or enrichie de pierreries où l'on avait les sept planètes et la terre au milieu."—*Historie de La Chine, par Martin,* page 76.

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was most successfully studied in the famous school of Alexandria. Here Euclid, the great geometrician, wrote a Treatise on the Sphere, entitled *The Phenomena*, which explained the most interesting parts of ancient astronomy; such as the right oblique ascension of the heavenly bodies, with the various other phenomena, which arise from the apparent diurnal revolution of the Primum Mobile. This work is supposed to be the first on the subject perfectly geometrical: it served long after as a model for other performances on the subject, and is still in existence, but very scarce.

Hipparchus, who flourished about two centuries after Euclid, and one before the Christian era, is said to have laid the foundation of Spherical Trigonometry. In succeeding ages it was improved by Ptolemy, Theodosius, and others; and much is ascribed to Geber, a learned Spaniard, who lived in the sixteenth century.

Baron Napier, however, made the most considerable improvements by his proposition of circular parts, and his invention of Logarithms,* and up to the present time many works of great merit have appeared in Europe. Yet the present, although the last, the author hopes will be found not the least useful.

* The author has discovered so simple a method of constructing Logarithms, that they may be calculated with as much facility as the most trifling question in common arithmetic, thus superseding the necessity of tables.

# SPHERICAL TRIGONOMETRY.

## DEFINITIONS.

### I.

A sphere is a solid, such that if it be cut by a plane in any position the section will be a circle.

or,

It is formed by the revolution of a semicircle about the diameter which remains fixed.

or,

It is a solid such that all lines drawn from a point within called the centre to the surface are equal to one another.

### II.

The circles cutting the sphere are divided into two kinds, the greater and lesser circles of the sphere: the greater passes through the centre, the less does not.

III.

The nearest distance between any two points on the sphere, is the arc of a great circle;—for such an arc being described with the greatest radius, is less curved than the arc of any small circle.

IV.

A spherical triangle is formed by three great circles on the surface of the sphere.

V.

A spherical angle is formed by the inclination of two great circles on the surface of the sphere meeting in a point, called the angular point.

VI.

Spherical triangles are also distinguished as right-angled, quadrantal and oblique:—thus when one of the angles is 90° it is called right-angled.

## VII.

Any obtuse-angled spherical triangle may be divided into two right-angled spherical triangles, by letting fall from any of the angular points on the opposite side a great circle, whose plane will be perpendicular to the plane of the base.

## OF RIGHT-ANGLED SPHERICAL TRIGONOMETRY.

Right-angled triangle spherical trigonometry may be divided into the six following cases:—

| | | |
|---|---|---|
| I. | When the hypothenuse and a leg, | are given together with the right angle. |
| II. | the hypothenuse and an angle, | |
| III. | a leg and its opposite angle, | |
| IV. | a leg and its adjacent angle, | |
| V. | two legs, | |
| VI. | two angles, | |

To find the remaining parts of the right-angled spherical triangle,

Place on the angular points of the annexed five-sided figure (its construction is simple) first the base, next the perpendicular, thirdly the complement of the angle opposite the base, fourthly the complement of the hypothenuse, fifthly the complement of the angle opposite the perpendicular, and in the centre radius as in the annexed diagram.

R, Radius: B, Base: P, Perpendicular: A, Complement of the angle opposite the base: H, complement of the hypothenuse: a, Complement of the angle opposite the perpendicular.